## Kinematic analysis

Everything we have done so far in this course has been about describing the 'here and now' of structural geology: where are structures and how are they oriented in the Earth's crust at the present day.
To understand the origin of structures we need to know how things changed during the formation of those structures - how things moved. When we study how things moved over geologic time, we are studying kinematics.

## 1. The basic movements

Deformation involves 4 types of movements

1. Translation, or change in position
2. Rotation or change in orientation
3. Dilation or change in area or volume
4. Distortion or change in shape

The first two are rigid body deformations. The last two together constitute non-rigid deformation or strain

## a) Translation

Translation is measured as a displacement. Any displacement has a distance and a direction. When we look at faults we will measure the displacement of one wall of a fault relative to another. The distance is measured in metres or kilometres, typically, whereas the direction is measured as a plunge and trend. Displacement is a vector quantitiy.

## b) Rotation

Rotation, or change in orientation, is typically measured in degrees, about a particular axis of rotation. A rotation is therefore also a vector quantity, because it has a magnitude and a direction.

A common kinematic problem involving rotation is to remove the effects of folding from some structure. For example, a sedimentologist may measure the orientation of a paleocurrent structure in folded strata, and wants to know the original direction of current flow.

This type of problem is most easily solved on the stereographic projection. As beds are 'unfolded', the paleocurrent directions they contain rotate along small circles on the stereonet. The small circles are centred on the fold axis.

## c) Dilation

Volume change is very difficult to measure in real rocks. However, under some circumstances it can be quantified. Most sedimentary rocks undergo some compaction as they are buried, because pore water is expelled. Compaction results in a negative dilation. Another common phenomenon
involving negative dilation is pressure solution in which some minerals in a rock are dissolved in response to stress.

Dilation is commonly measured as
$\Delta=\frac{V-V_{0}}{V_{0}}$ where V is the present volume and $\mathrm{V}_{0}$ is the original volume.
Dilation is a scalar quantity (it just has a magnitude).

## d) Distortion

Distortion is by far the most complicated type of deformation to measure. When rocks are distorted they typically get longer in some directions and shorter in others. Also, angles change in distortion. Because of this, strain cannot be represented by a scalar or a vector. It is a more complicated quantity that is called a tensor.

## Changes in length (longitudinal strain)

There are two ways to measure change in length
Elongation (sometimes extension) $e=\left(l-l_{0}\right) / l_{0}$

$$
\text { Stretch } S=l / l_{0}=1+e
$$

where original length is $l_{0}$ and new length is $l$
The important thing to remember is that in strained rocks the elongation varies with direction: typically some lines will have got longer and others will have got shorter.

## Changes in angles (shear strain)

To measure change in angles we look at two lines that were originally perpendicular.

If the change in angle is $\psi$ then
Shear strain $\gamma=\tan \psi$


There are other methods used to measure shear strain in advanced studies, so the above definition is sometimes qualified as engineering shear strain because it is the measure allegedly most used by engineers.

## 2. Strain

## a) Heterogeneous strain and homogeneous strain

Strain can vary from place to place in rocks. We distinguish a special case:

Homogenous strain: strain that is the same everywhere within a body of rock. In homogeneous strain, straight lines remain straight, parallel lines remain parallel, circles are deformed into ellipses.

Strain that is not homogeneous is heterogenous. In heterogenous strain, straight lines can be bent, and lines that were initially parallel are rotated by different amounts, becoming non-parallel.

Heterogeneous strain is difficult to deal with mathematically. However, if we look at a very small region of a heterogeneously strained rock, it can often be treated in the same way as a homogeneous strain. The strain at a point in a heterogeneously strained rock follows the same mathematical rules as homogenous strain.

## b) Strain ellipse

The strain ellipse is a convenient way to represent a state of homogenous strain, or the strain at a point, on a two-dimensional surface. The strain ellipse is the shape of a deformed circle that originally had unit radius.

The radius of the strain ellipse in any direction is equal to the stretch S in that direction. The strain ellipse is thus a good way to represent the variation of longitudinal strain with direction.

## c) Strain ellipsoid

The strain ellipsoid is a convenient way to represent a state of homogenous strain, or the strain at a point, in three dimensions. The strain ellipsoid is the shape of a deformed sphere that originally had unit radius.

The radius of the strain ellipsoid in any direction is equal to the stretch S in that direction. In 3-D, he strain ellipsoid is thus a good way to represent the variation of longitudinal strain with direction.

## d) Strain axes

A strain ellipse has two lines that are special. They represent the maximum and minimum stretches, called $\mathbf{S}_{\mathbf{1}}$ and $\mathbf{S}_{\mathbf{3}}$ respectively. These lines are strain axes. (The strain axes are sometimes called X and Z ; however, coordinate axes on a map or cross-section are sometimes $x$ and z. Strain axes don't necessarily coincide with any special direction on a map, so this terminology may cause confusion.)

Strain axes have some other special properties. They are always at right angles to each other, and they also represent lines of zero shear strain. This means that they were perpendicular before deformation started too. (However, during deformation they may have diverged from this perpendicular relationship and then come back to it!).

The strain ratio is a convenient measure of the amount of distortion in 2D. The strain ratio is the ratio between the long axis and the short axis of the strain ellipse:

## Strain ratio $R_{S}=S_{1} / S_{3}$

A strain ellipsoid has three lines that are special. They represent the maximum and minimum stretches, called $\mathbf{S}_{\mathbf{1}}$ and $\mathbf{S}_{\mathbf{3}}$ respectively, and a third, intermediate axis of intermediate stretch $\mathbf{S}_{\mathbf{2}}$, that is mutually perpendicular to the other two. These lines are strain axes. (The strain axes are sometimes called $\mathrm{X}, \mathrm{Z}$ and Y .)

They have some other special properties. They are always at right angles to each other, and they also represent lines of zero shear strain. This means that they were perpendicular before deformation started too. (However, during deformation they may have diverged from this perpendicular relationship and then come back to it!).

## 3. Deformation histories

## a) Rotational and non-rotational deformation

All the above measures have concerned just strain. However, if we look at the whole deformation picture, we may see situations where rotation has gone on at the same time as strain. Under these circumstances it's helpful to look at the behaviour of the strain axes.

If the strain axes have the same orientation as they did before deformation started, then the deformation is non-rotational (sometimes irrotational).
If the strain axes have rotated during deformation, then the deformation is described as rotational.

## b) Finite and infinitesimal deformation

Once we start looking at rotation, it's difficult to avoid discussing strain history too. When we look at a deformed rock what we see is the product of a whole history of deformation. That end product is called the finite deformation and the strain part of the deformation is the finite strain.

In more detailed analyses of strain, we may be interested in all the tiny increments of strain that have contributed to the final picture. Each one is called an incremental strain.

The infinitesimal strain is the end product of this type of thinking. The strain history is thought of as being made up of an infinite number of infinitesimal strain increments. (The general idea should be familiar to anyone who has taken calculus.) This idea of infinitesimal strain becomes
important if we look at strain rates. Strain rates are typically measured in units of strains per second or $\mathrm{s}^{-1}$. In geologically reasonable situations, the amount of strain that occurs in a second is almost infinitesimal. Typical ductile strain rates in the Earth's crust are thought to be between $10^{-12}$ and $10^{-15} \mathrm{~s}^{-1}$.

## c) Coaxial and non-coaxial deformation

The idea of strain history also allows us to refine our ideas about rotational and non-rotational strain.

If every step of the strain history is non-rotational, that means that the strain axes were fixed to the same lines of particles (material lines) throughout deformation. This special type of deformation is called coaxial deformation. It is also known as pure strain, because no rotation is involved.
(You may also encounter the term pure shear which means almost the same thing. Pure shear is a pure strain in which there is also no volume change.)

If the strain axes rotate during deformation from one row of rock particles to a different one. then the deformation is non-coaxial.

There is a special type of non-coaxial deformation that occurs in shear zones, particularly, that has similar kinematic characteristics to a sliding deck of cards. It is called simple shear. In simple shear, all particles move parallel to a particular line, along which the stretch is 1 . It is important to note that there are lots of other types of non-coaxial deformation; simple shear is just one type.

